MV16 - Summer Work – Algebra Questions

Get Started….

Q1.

$x = 0.7$

Work out the value of $\dfrac{(x+1)^2}{2x}$

Write down all the figures on your calculator display.

Basic Algebra

Q2.

(a) Expand $7(x + 5)$

(b) Expand $3y(4y - 3)$

(c) Expand and simplify $(t + 2)(t + 4)$

Q3.

(a) Expand and simplify $3(x + 4) + 2(5x - 1)$

(b) Expand and simplify $(2x + 1)(x - 4)$

(c) Factorise completely $6y^2 - 9xy$

Quadratics

Q4.

Factorise $x^2 + 3x - 4$

Q4.

Solve, by factorising, the equation $8x^2 - 30x - 27 = 0$

Q6.

Factorise $x^2 - 121$

Q7.

Solve $x^2 - 17x - 56 = 0$
Give your solutions correct to 2 decimal places.

Q8.

Alison is using the quadratic formula to solve a quadratic equation.
She substitutes values into the formula and correctly gets

$$x = \frac{-7 \pm \sqrt{49 - 32}}{4}$$

Work out the quadratic equation that Alison is solving.
Give your answer in the form $ax^2 + bx + c = 0$, where a, b and c are integers.

Simultaneous Equations (Linear)

Q9.

Solve the simultaneous equations

$$3x + 10y = 7$$
$$x - 4y = 6$$

Q10.

Solve the simultaneous equations

$$3x - 2y = 7$$
$$7x + 2y = 13$$

Q11.

Solve the simultaneous equations

$$2x + 3y = 10$$
$$4x - y = -1$$

Indices

Q12.

(a) Simplify $(p^3)^2$

(b) Simplify $\frac{t^8}{t^3}$

(c) Simplify $n^4 \div n^{\frac{1}{2}}$

$2^3 \times 2^n = 2^9$

(d) Work out the value of *n*.

$2x^3 = 128$

(e) Work out the value of *x*.

Changing the subject of a formula

Q13.

Make *p* the subject of the formula $y = 3p^2 - 4$

Q14.

Make *t* the subject of the formula $2(d - t) = 4t + 7$

Q15.

$$q = \frac{p}{r} + s$$

Make *p* the subject of this formula.

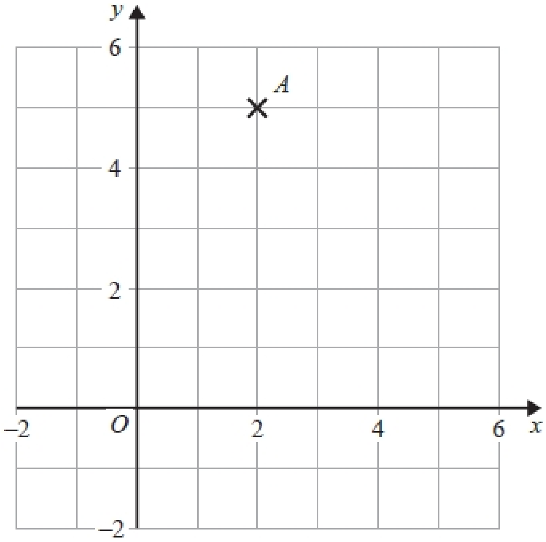
Q16.

Make t the subject of the formula $y = \frac{t}{3} - 2a$

Straight Line Graphs

Q17.

Find an equation of the straight line with gradient 3 that passes through point A.



Completing the Square (Quadratics)

Q18.

Write $x^2 + 6x - 7$ in the form $(x + a)^2 + b$ where a and b are integers.

Q19.

The expression $x^2 - 8x + 6$ can be written in the form $(x - p)^2 + q$ for all values of x.

(a) Find the value of p and the value of q.

The graph of $y = x^2 - 8x + 6$ has a minimum point.

(b) Write down the coordinates of this point.

(............................ , ...)

Mixed Bag

Q20.

$-5 < y \leq 0$

y is an integer.

(a) Write down all the possible values of y.

(b) Solve $6(x - 2) > 15$

Q21.

(a) Expand and simplify $(y - 2)(y - 5)$

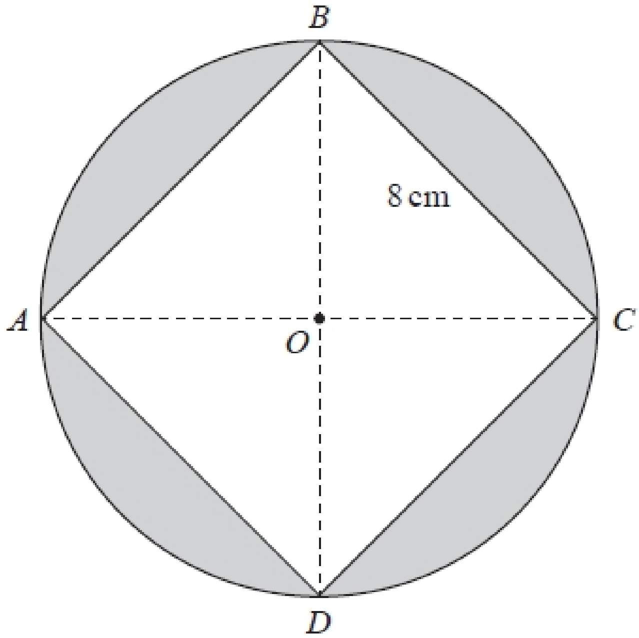
(b) Prove algebraically that

$$(2n + 1)^2 - (2n + 1) \text{ is an even number}$$

for all positive integer values of n.

Q22.

The diagram shows a square *ABCD* of side 8 cm inside a circle, centre *O*.
The vertices of the square lie on the circle.



Work out the total area of the four shaded segments.

Give your answer correct to 3 significant figures.

Q23.

Simplify fully $(\sqrt{a} + \sqrt{4b})(\sqrt{a} - 2\sqrt{b})$

Q24.

(a) Expand and simplify $(x + 2)(2x - 3)(3x + 1)$

Q25.

Solve the simultaneous equations

$$x^2 + y^2 = 25$$
$$y = 2x + 5$$

Algebraic Fractions

Q26.

Simplify $\dfrac{x+1}{2} + \dfrac{x+3}{3}$

Q27.

Solve $\dfrac{x+1}{2} + \dfrac{2x-1}{3} = \dfrac{5}{6}$

Q28.

Solve $\dfrac{3x-2}{4} - \dfrac{2x+5}{3} = \dfrac{1-x}{6}$

Functions

Q29.

f and g are functions such that

$$f(x) = 3x^2 \quad \text{and} \quad g(x) = \frac{1}{x-2}$$

Find gf(4).
Give your answer as a fraction.

Q30.

$f(x) = x^3$
$g(x) = 4x - 1$

(a) Find fg(2)

$h(x) = fg(x)$

(b) Find an expression for $h^{-1}(x)$

Some things to think about…..

A room is a cuboid - 5m x 4m x 2m. A spider in one corner of the room (on the floor) wants to catch a fly at the opposite corner of the room (on the ceiling). Assuming the fly is not clever enough to fly away and that the spider can climb walls and walk on the ceiling, what is the shortest route that the spider can take to catch the fly?

Gauss (1777-1855) was a famous mathematician. When he was at primary school, his teacher instructed the class to add the integers (whole numbers) from 1 to 100. Thinking this might occupy the class for a while; the teacher was surprised when Gauss responded with the correct answer almost immediately. What trick did Gauss hit upon?

<u>The proof that √2 is irrational</u>

We assume that every number is the product of primes (*AKA*: "The Fundamental Theorem of Arithmetic"). Now, let √2 be rational, such that $(p/q)^2 = 2$ for integers p and q. Then $p^2 = 2q^2$. Now factorised into primes, p^2 is factored into the same set of primes as p taken twice. Therefore p^2 has an *even* number of prime factors. So does q^2 for the same reason. Therefore $2q^2$ has an *odd* number of prime factors. So the same number has an *even* number and an *odd* number of prime factors, which is a *contradiction*. We must therefore reject the original proposition that √2 is rational. It *must* be irrational.